fourth GRADE

2018-2019 Curriculum Guide

October 29- January 11

<u>Eureka</u>

Module 3: *Multi-Digit Multiplication and Division*

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Table of Contents

Module 3 Performance Overview

- Students begin Topic A by investigating the formulas for area and perimeter. They use those formulas to solve for area and perimeter and to find the measurements of unknown lengths and widths. Students create diagrams to represent these problems as well as write equations with symbols for the unknown quantities.

- In Topic B, students examine multiplication patterns when multiplying by 10, 100, and 1,000. Teachers also continue using the phrase "____ is ____ times as much as ____" (e.g., 120 is 3 times as much as 40). This carries forward multiplicative comparison in the context of both calculations and word problems. In preparation for two-digit by two-digit multiplication, students practice the new complexity of multiplying two two-digit multiples of 10.

- Students begin in Topic C decomposing numbers into base ten units in order to find products of single-digit by multi-digit numbers. Students practice multiplying by using models before being introduced to the standard algorithm.

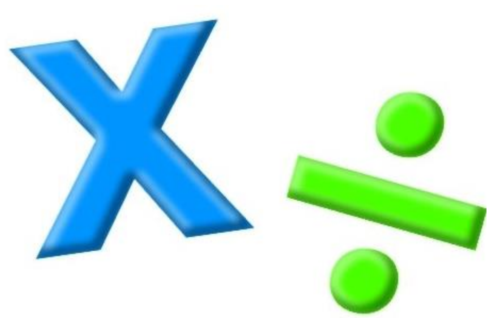
- Topic D gives students the opportunity to apply their new multiplication skills. Students extend their work with multiplicative comparison from Topic A to solve real-world problems. Students also use a combination of addition, subtraction, and multiplication to solve multi-step problems.

- Students focus on interpreting the remainder within division problems both in word problems and long division. Students apply this simple idea to divide two-digit numbers unit by unit: dividing the tens units first, finding the remainder (the number of tens unable to be divided), and decomposing remaining tens into ones to then be divided.

- In Topic F, armed with an understanding of remainders, students explore factors, multiples, and prime and composite numbers within 100. Students gain valuable insights into patterns of divisibility as they test for primes and find factors and multiples.

- Topic G extends to division with three- and four-digit dividends using place value understanding. Students begin the topic by connecting multiplication of 10, 100, and 1,000 by single-digit numbers from Topic B to division of multiples of 10, 100, and 1,000.

- Module 3 closes with Topic H as students multiply two-digit by two-digit numbers. Students use the area model to represent and solve the multiplication of two-digit multiples of 10 by two-digit numbers using a place value chart.



Module 3: *Multi-Digit Multiplication and Division*

Pacing: October 29- January 11 **40 Days**		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
Topic A: Multiplicative Comparisons Word Problems	Lesson 1	Investigate and use the formulas for area and perimeter of rectangles. https://www.youtube.com/watch?v
	Lesson 2	Solve multiplicative comparison word problems by applying the area and perimeter formulas. https://www.youtube.com/watch?v
	Lesson 3	Demonstrate understanding of area and perimeter formulas by solving multi-step real world problems. https://www.youtube.com/watch?v
Topic B: Multiplication by 10, 100, and 1,000	Lesson 4	Interpret and represent patterns when multiplying by 10, 100, and 1,000 in arrays and numerically. https://www.youtube.com/watch?v
	Lesson 5	Multiply multiples of 10, 100, and 1,000 by single digits, recognizing patterns. https://www.youtube.com/watch?v
	Lesson 6	Multiply two-digit multiples of 10 by two-digit multiples of 10 with the area model. https://www.youtube.com/watch?v
Topic C: Multiplication of up to Four Digits by Single-Digit Numbers	Lesson 7	Use place value disks to represent two-digit by one-digit multiplication. https://www.youtube.com/watch?v
	Lesson 8	Extend the use of place value disks to represent three- and four-digit by one-digit multiplication. https://www.youtube.com/watch?v
	Lesson 9	Multiply three- and four-digit numbers by one-digit numbers applying the standard algorithm. https://www.youtube.com/watch?v
	Lesson 11	Connect the area model and the partial products method to the standard algorithm. https://www.youtube.com/watch?v
Topic D: Multiplication Word Problems	Lesson 12	Solve two-step word problems, including multiplicative comparison. https://www.youtube.com/watch?v
	Lesson 13	Use multiplication, addition, or subtraction to solve multi-step word problems. https://www.youtube.com/watch?v
Mid- Module Assessment		

November 19-20, 2018		
Topic E: Division of Tens and Ones with Successive Remainders	Lesson 14	Solve division word problems with remainders. https://www.youtube.com/watch?v
	Lesson 15	Understand and solve division problems with a remainder using the array and area models. https://www.youtube.com/watch?v
	Lesson 16	Understand and solve two-digit dividend division problems with a remainder in the ones place by using number disks. https://www.youtube.com/watch?v
	Lesson 17	Represent and solve division problems requiring decomposing a remainder in the tens. https://www.youtube.com/watch?v
	Lesson 18	Find whole number quotients and remainders. https://www.youtube.com/watch?v
	Lesson 19	Explain remainders by using place value understanding and models. https://www.youtube.com/watch?v
	Lesson 20	Solve division problems without remainders using the area model. https://www.youtube.com/watch?v
Topic F: Reasoning with Divisibility	Lesson 22	Find factor pairs for numbers to 100 and use understanding of factors to define prime and composite. https://www.youtube.com/watch?v
	Lesson 23	Use division and the associative property to test for factors and observe patterns. https://www.youtube.com/watch?v
	Lesson 24	Determine whether a whole number is a multiple of another number. https://www.youtube.com/watch?v
	Lesson 25	Explore properties of prime and composite numbers to 100 by using multiples. https://www.youtube.com/watch?v
Topic G: Division of Thousands, Hundreds, Tens, and Ones	Lesson 26	Divide multiples of 10, 100, and 1,000 by single-digit numbers. https://www.youtube.com/watch?v
	Lesson 27	Represent and solve division problems with up to a three-digit dividend numerically and with number disks requiring decomposing a remainder in the hundreds place. https://www.youtube.com/watch?v

	Lesson 28	Represent and solve three-digit dividend division with divisors of 2, 3, 4, and 5 numerically. https://www.youtube.com/watch?v
	Lesson 29	Represent numerically four-digit dividend division with divisors of 2, 3, 4, and 5, decomposing a remainder up to three times. https://www.youtube.com/watch?v=NiMbVsLuCpU&index=29&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr
	Lesson 30-33	Solve division problems with a zero in the dividend or with a zero in the quotient. https://www.youtube.com/watch?v=GXY9unJJHWA&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr&index=30 Explain the connection of the area model of division to the long division algorithm for three- and four-digit dividends. https://www.youtube.com/watch?v=dZ2_9xRGf-Y&index=33&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr
	Lesson 31	Interpret division word problems as either *number of groups unknown* or *group size unknown*. https://www.youtube.com/watch?v=VTW2tt0odgA&index=31&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr
	Lesson 32	Interpret and find whole number quotients and remainders to solve one-step division word problems with larger divisors of 6, 7, 8, and 9. https://www.youtube.com/watch?v=20l35vvqcNU&index=32&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr
Topic H: Multiplication of Two-Digit by Two-Digit Numbers	Lesson 34	Multiply two-digit multiples of 10 by two-digit numbers using a place value chart. https://www.youtube.com/watch?v=GFqUpELGJ-g&index=34&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr
	Lesson 35	Multiply two-digit multiples of 10 by two-digit numbers using the area model. https://www.youtube.com/watch?v=kUUoBNMSy4A&index=35&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr
	Lesson 36	Multiply two-digit by two-digit numbers using four partial products. https://www.youtube.com/watch?v=ujdcH5X1vgY&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr&index=36
	Lesson 37	Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication. https://www.youtube.com/watch?v=T7oYai_WPqs&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr&index=37
	Lesson 38	Transition from four partial products to the standard algorithm for two-digit by two-digit multiplication. https://www.youtube.com/watch?v=N3OUYK7JPzY&list=PLvolZqLMhJmne18B7_qMvEOnxdCusfXjr&index=38
End of Module Assessment **January 10-11, 2019**		

NJSLS Standards:

4.OA.1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations. A multiplicative comparison is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "a is n times as much as b").

- Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times. Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

Example:
5 x 8 = 40. Sally is five years old. Her mom is eight times older. How old is Sally's Mom? 5 x 5 = 25
Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

- Utilize the properties and patterns of multiplication (including the commutative, associative, and identity properties). Identify and verbalize which quantity is being multiplied and which number tells by how many times. Explore the meaning of the two factors in comparison multiplication problems. Practice writing and identifying equations and statements for multiplicative comparisons. Use manipulatives to represent how many times greater the area of one shape is than another, such as pattern blocks. Provide contextualized situations which make use of diagrams, a table, and equations.

Example: There were thirty-two adults and four children in line at a movie theater. How many times more adults were in the line than children?

- Utilize multiplicative thinking, known multiples, and the meaning of each factor/product. Interpret diagrams that focus on unmeasured multiplicative relationships. Explain multiplication equations to represent comparisons. Promote words like "doubling" and "tripling" to connect to "two times as much" and "three times as much" to introduce multiplicative relationships. Distinguish multiplicative comparison from the additive comparison.

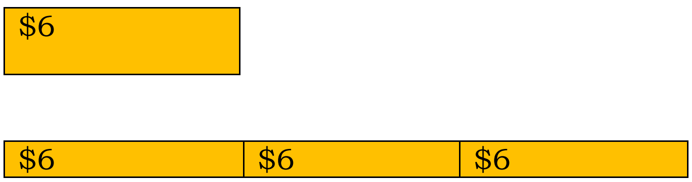
- A situation that can be represented by multiplication has an element that represents the scalar and an element that represents the quantity to which the scalar applies. (NCTM, Essential Understanding, 2011).

- A multiplicative comparison involves a constant increase that x is more times or x times less; whereas an additive comparison only involves determining how many more than or how many less than another set.

- One of the factors in multiplication indicates the number of objects in a group and the other factor indicates the number of groups. In the multiplicative expression A x B, A can be defined as a scaling factor. (NCTM, Essential Understanding, 2011).

4.OA.2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

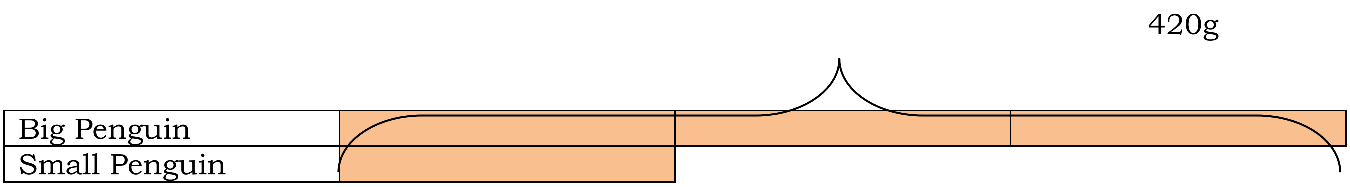
- This standard calls for students to translate comparative situations into equations with an unknown and solve. Students need many opportunities to solve contextual problems.
- In an additive comparison, the underlying question is *what amount would be added to one quantity* in order to result in the other. In a multiplicative comparison, the underlying question is *what factor would multiply one quantity* in order to result in the other.



3 x *B*=*R*
3 x $6 = $18

A tape diagram used to solve a Compare problem:

A big penguin will eat 3 times as much fish as a small penguin. The big penguin will eat 420 grams of fish. All together, how much will the two penguins eat?

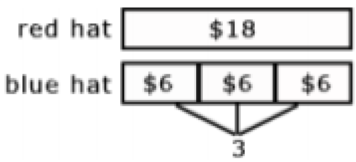


B=number of grams the big penguin eats
S=number of grams the small penguin eats
3 x S=B
3 x S=420
S=140
S + B=140 + 420 =560

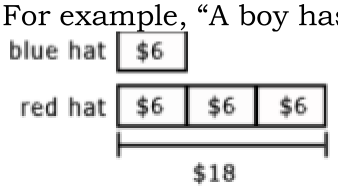
- Identify when multiplication or division must be used in a multiplicative comparison question. For example, when "5 times more" indicates multiplication and "5 times less" indicates division. Identify what amount would be added to or subtracted from one quantity in order to result in the other. Recognize the inverse relationship between multiplication and division, and determine that division can be used to solve comparison multiplication problems when either group size or the scaling factor is provided. Tape diagrams can be used as a strategy in solving problems with multiplicative comparisons.

<u>*There are three kinds of multiplicative comparison word problems:*</u>
Product unknown comparisons.

For example, “A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?”



Set size unknown comparisons,
For example, “A boy has $12 and each red hat costs $3. How many hats can the boy buy?”



Multiplier unknown comparisons,
For example, “A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat?”

- Students should be able to distinguish multiplicative comparison from additive comparisons. Assess students’ interpretations of a model in use to determine whether their view demonstrates mathematical understanding. Translate comparative situations into equations. Represent problems with drawings and equations, using a symbol for the unknown number. *Find evidence in a word problem to help develop an equation and support the operation chosen to solve.*

4.OA.3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

- The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems using all four operations.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?
Some typical estimation strategies for this problem:

Student 1	**Student 2**	**Student 3**
I first thought about 267 and	I first thought about 194. It is	I rounded 267 to 300. I round-

34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	ed 194 to 200. I rounded 34 to 30. When I added 300, 200 and 30, I know my answer will be about 530. The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550).

- Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies.

Estimation strategies include, but are not limited to:

- front-end estimation with adjusting (using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts),
- clustering around an average (when the values are close together an average value is selected and multiplied by the number of values to determine an estimate),
- rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding affected the original values),
- using friendly or compatible numbers such as factors (students seek to fit numbers together e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000),
- using benchmark numbers that are easy to compute (students select close whole numbers for fractions or decimals to determine an estimate).

4.OA.4	Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1– 100 is prime or composite.

- This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers.

- Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17.

- Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

- A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite.

- Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

Students investigate whether numbers are prime or composite by

- building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g. 7 can be made into only 2 rectangles, 1 x 7 and 7 x 1, therefore it is a prime number)

- finding factors of the number Students should understand the process of finding factor pairs so they can do this for any number 1 – 100

Example: Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12. Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

To determine if a number between1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- all even numbers are multiples of 2
- all even numbers that can be halved twice (with a whole number result) are multiples of
- all numbers ending in 0 or 5 are multiples of 5

4.NBT.5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models

- Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication.

- The commutative and associative properties of multiplication ensure flexibility in computations with whole numbers and provide justifications for sequences of computations with them

- Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. ***Use of the standard algorithm for multiplication is an expectation in the 5th grade***

- Another part of understanding general base-ten methods for multi-digit multiplication is understanding the role played by the distributive property. This allows numbers to be decomposed into base-ten units, products of the units to be computed, and then combined. By decomposing the factors into like base-ten units and applying the distributive property, multiplication computations are reduced to single-digit multiplications and products of numbers with multiples of 10, of 100, and of 1000.

- Students can connect diagrams of areas or arrays to numerical work to develop understanding of general base-ten multiplication methods. Computing products of two two-digit numbers requires using the distributive property several times when the factors are decomposed into base-ten units.

Example:
36 X 94=(30 + 6) X (90+4) =(30+6) X 90(30 +6) X 4 = 30 X90 + 6 X 90 + 30 X 4 + 6 X 4

Computation of 36 x 94 connected with an area model.

	90	+4
30	30 X 90= 3 tens 9 tens= 27 hundreds= 2,700	30 X 4= 3 tens X 4= 12 tens 120
6	6 X 90= 6 X 9 tens= 54 tens= 540	6 X 4=24

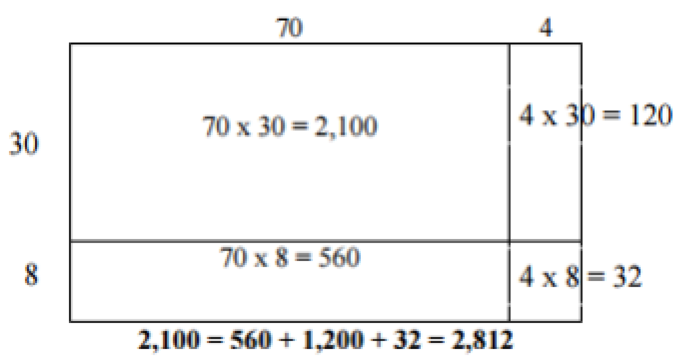
The products of like base-ten units are shown as parts of a rectangular region.

- This standard calls for students to multiply numbers using a variety of strategies.

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

Student 1	Student 2	Student 3
25 x12 I broke 12 up into 10 and 2 25 x 10 = 250 25 x 2 = 50 250 +50 = 300	25 x 12 I broke 25 up into 5 groups of 5 5 x 12 = 60 I have 5 groups of 5 in 25 60 x 5 = 300	25 x 12 I doubled 25 and cut 12 in half to get 50 x 6 50 x 6 = 300

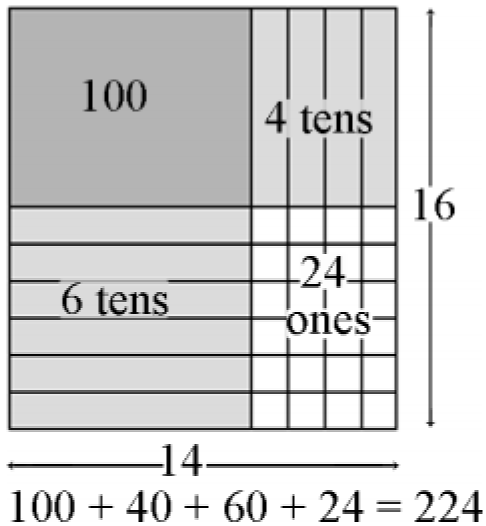
Example:
What would an array area model of 74 x 38 look like?



8 x 4 =32 30 x4=120 70 x 30 = 2,100 70 x 8 = 560 2,100+ 560 + 120 + 32 = 2,812

The area model below shows the partial products.

14 x 16 = 224

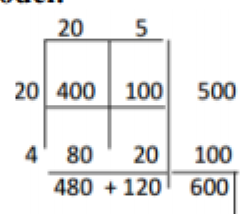


Using the area model, students first verbalize their understanding:

- 10 x 10 is 100
- 4 x 10 is 40
- 10 x 6 is 60, and
- 4 x 6 is 24.

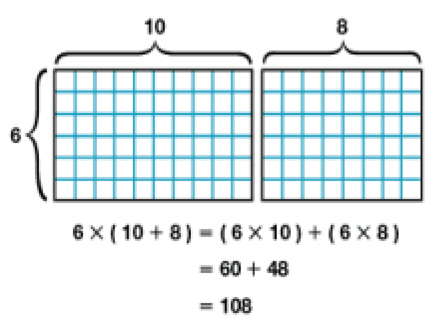
Base Ten Blocks:
25 x 24 = 400 (20 x 20) +100 (20 x 5) +80 (4 x 20) +20 (4 x 5)= 600

Matrix Model:



Equation: 5 x 241= 5(200+40+1)

Rectangular Array:



- Eliminate using “placeholder” because in the multiplication algorithm , the 0 does more than just hold a place; it shows the value of the ones place is 0 in multiplication by tens.

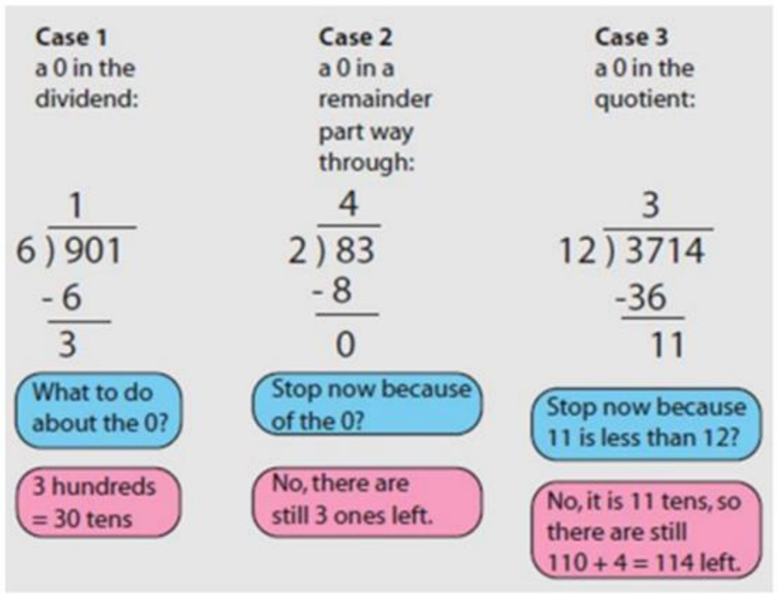
4.NBT.6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- ***Standard algorithm for division is taught in 6th grade,*** but when referring to how many times a divisor goes into dividend place value has to be emphasized avoiding statements such as " How many times does 2 go into 2?" when dividing 24 ÷ 2 =.
- In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context. General methods for computing quotients of multi-digit numbers and one-digit numbers rely on the same understandings as for multiplication.
- One component is quotients of multiples of 10, 100, or 1000 and one-digit numbers. For example, 42 ÷ 6 is related to 420 ÷ 6 and 4200 ÷ 6. Students can draw on their work with multiplication and they can also reason that 4200 6 means partitioning 42 hundreds into 6 equal groups, so there are 7 hundreds in each group.
- Another component of understanding general methods for multi-digit division computation is the idea of decomposing the dividend into like base-ten units and finding the quotient unit by unit, starting with the largest unit and continuing on to smaller units. As with multiplication, this relies on the distributive property. This can be viewed as finding the side length of a rectangle (the divisor is the length of the other side) or as allocating objects (the divisor is the number of groups).
- Multi-digit division requires working with remainders. In preparation for working with remainders, students can compute sums of a product and a number, such as 4 x 8 + 3. In multi-digit division, students will need to find the greatest multiple less than a given number.

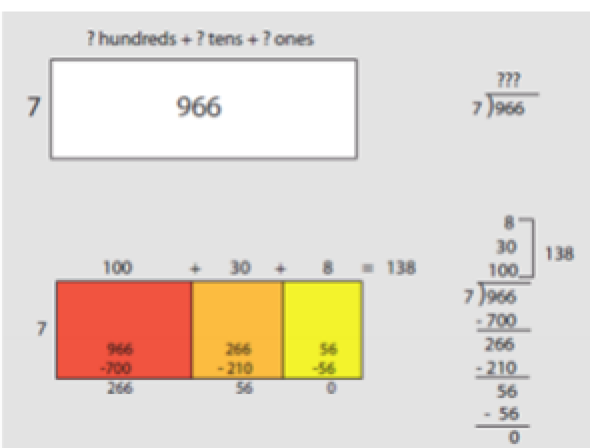
Example:
When dividing by 6, the greatest multiple of 6 less than 50 is 6 x 8 = 48. Students can think of these "greatest multiples" in terms of putting objects into groups. For example, when 50 objects are shared among 6 groups, the largest whole number of objects that can be put in each group is 8, and 2 objects are left over. (Or when 50 objects are allocated into groups of 6, the largest whole number of groups that can be made is 8, and 2 objects are left over.) The equation 6 x 8 + 2 = 50 (or 8 x 6 + 2 = 50) corresponds with this situation.

- Cases involving 0 in division may require special attention

Cases Involving 0 In Division

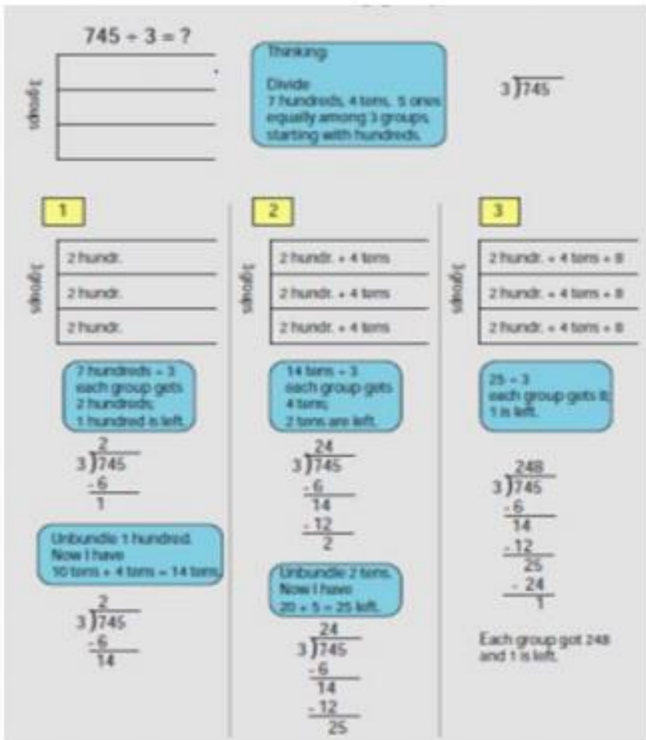


Cases involving finding missing side length



- 966 ÷ 7 is viewed as finding the unknown side length of a rectangular region with an area of 966 square units and a side of the length 7 units. The amount of hundreds is found, the tens, then ones. This yields a decomposition into three regions of dimensions 7 by 100, 7 by 30, ad 7 by 8. It can be connected with the decomposition of 966 as 7 x 100 +7 x 30+7 x8. By the distributive property, this is 7 x(100+30+8), so the unknown side length is 138. In the recording on the right, amounts of hundreds,, tens and ones are represented by numbers rather tha by digits, e.g. 700 instead of 7.

Division As Finding Group Size



- 745 ÷ 3 can be viewed as allocating 745 objects bundled in 7 hundreds, 4 tens, and 3 ones equally among 3 groups. In step 1, the 2 indicates that each group got 2 hundreds, the 6 is the number of hundreds allocated. After Step 1, the remaining hundred is decomposed as 10 tens and combined with the 4 tens (in 745) to make 14 tens.

Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- Using Base 10 Blocks: Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.

- Using Place Value: 260 ÷ 4 = (200 ÷ 4) + (60 ÷ 4)

- Using Multiplication: 4 x 50 = 200, 4 x 10 = 40, 4 x 5 = 20; 50 + 10 + 5 = 65; so 260 ÷ 4 = 65 This standard calls for students to explore division through various strategies.

Example:
There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

- Using an Open Array or Area Model After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

Example:
592 ÷ 8 =

Student 1	Student 2	Student 3
592 divided by 8 There are 70 8's in 560 592 - 560 = 32 There are 4 8's in 32 70 + 4 = 74	592-400 50 192-160 20 32-32 4 0 592 divided by 8 I know that 10 8's is 80 If I take out 50 8's that is 400 592 - 400 = 192 I can take out 20 more 8's which is 160 192 - 160 = 32 8 goes into 32 4 times I have none left I took out 50, then 20 more, then 4 more That's 74	I want to get to 592 8 x 25 = 200 8 x 25 = 200 8 x 25 = 200 200 + 200 + 200 = 600 600 - 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams

4.MD.3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor

- Know and apply the formula for area (L x W) and express the answer in square units. Know and apply the formula for perimeter: 2L + 2W or 2(L + W) and express the answer in linear units. The formulas can be used to find unknown factor. Ex. (8 x W) = 24 sq. units.
- Communicate understanding of formula and justify why it works. Formulas should be developed with students through experience not just memorization.

Common multiplication and division situations. [1]

	UNKNOWN PRODUCT	GROUP SIZE UNKNOWN ("HOW MANY IN EACH GROUP?" DIVISION)	NUMBER OF GROUPS UNKNOWN ("HOW MANY GROUPS?" DIVISION)
	3 x 6 = ?	**3 x ? = 18**, and **18 ÷ 3 = ?**	**? x 6 = 18**, and **18 ÷ 6 = ?**
EQUAL GROUPS	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example.* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example.* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
ARRAYS [2], AREA [3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example.* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example.* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example.* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
COMPARE	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example.* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example.* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example.* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
GENERAL	**a x b = ?**	**a x ? = p** and **p ÷ a = ?**	**? x b = p**, and **p ÷ b = ?**

[1] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[2] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[3] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Module 3 Assessment / Authentic Assessment Recommended Framework			
Assessment	**CCSS**	**Estimated Time**	**Format**
Eureka Math Module 3: Multi-Digit Multiplication and Division			
Authentic Assessment #1	4.NBT.6	30 mins	Individual
Authentic Assessment #2	4.OA.1 4.NBT.5	30 mins	Individual
Optional Mid Module Assessment	4.OA.1-4 4.NBT.5-6 4.MD.3	1 Block	Individual
Optional End of Module Assessment	4.OA.1-4 4.NBT.5-6 4.MD.3	1 Block	Individual
Grade 4 Interim Assessment 1 (IREADY) ***10/29-11/12***	4.OA.3 4.NBT.1-4	1 Block	Individual

Fourth Grade Ideal Math Block

Fluency: Whole Group
Sprints, Counting, Whiteboard Exchange

Application Problem: Whole Group
Provides HANDS-ON work to allow children to ACT OUT or ENGAGE ACTIVELY with the new MATH IDEA

Concept Development: Individual/partner/whole
Instruction & Strategic Problem Set Questions

Student Debrief: Whole Group
Exit Ticket: Independent

50-60 min.

CENTERS/STATIONS:
Pairs / Small Group/ Individual
DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

20-30 min.

M: Meet with the teacher	A: Application/ Problem Solving	T: Technology	H: Hands on Activities

Eureka Lesson Structure:

Fluency:

- Sprints
- Counting : Can start at numbers other than 0 or 1 and might include supportive concrete material or visual models
- Whiteboard Exchange

Application Problem:

- Engage students in using the RDW Process
- Sequence problems from simple to complex and adjust based on students' responses
- Facilitate share and critique of various explanations, representations, and/or examples.

Concept Development: (largest chunk of time)

Instruction:

- Maintain overall alignment with the objectives and suggested pacing and structure.
- Use of tools, precise mathematical language, and/or models
- Balance teacher talk with opportunities for peer share and/or collaboration
- Generate next steps by watching and listening for understanding

Problem Set: (Individual, partner, or group)

- Allow for independent practice and productive struggle
- Assign problems strategically to differentiate practice as needed
- Create and assign remedial sequences as needed

Student Debrief:

- Elicit students thinking, prompt reflection, and promote metacognition through student centered discussion
- Culminate with students' verbal articulation of their learning for the day
- Close with completion of the daily Exit Ticket (opportunity for informal assessment that guides effective preparation of subsequent lessons) as needed.

PARCC Assessment Evidence/Clarification Statements			
CCSS	**Evidence Statement**	**Clarification**	**MP**
4.OA.1-1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5	• Tasks have "thin context" or no context.	MP.2,4
4.OA.1-2	Represent verbal statements of multiplicative comparisons as multiplication equations.	• Tasks have "thin context" or no context.	MP 2,4
4.OA.2.	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison	• See the OA Progression document, especially p. 29 and Table 2, Common Multiplication and Division situations on page 89 of CCSSM. • Tasks sample equally the situations in the third row of Table 2 on page 89 of CCSSM	MP 1,4, 5
4.OA.3-1	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations.	• Assessing reasonableness of answer is not assessed here. • Tasks do not involve interpreting remainders.	MP.1,2,7
4.OA.3-2	Solve multi-step word problems posed with whole numbers and having whole-number answers using the four operations, in which remainders must be interpreted	• Assessing reasonableness of answer is not assessed here. • Tasks involve interpreting remainders. • See p. 30 of the OA Progression document. • Multi-step problems must have at least 3 steps.	MP 1,2,4,7
4.OA.4-1	Find all factor pairs for a whole number in the range 1–100.		MP.7
4.OA.4-2	Recognize that a whole number is a multiple of each of its factors. -		MP 2
4.OA.4-3	Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. -		MP 8
4.OA.4-4	Determine whether a given whole number in the range 1–100 is prime or composite.		MP 7,8

4.NBT.5 -1	Multiply a whole number of up to four digits by a one-digit whole number using strategies based on place value and the properties of operations.	. i) Tasks do not have a context. ii) For the illustrate/explain aspect of 4.NBT.5, see 4.C.1-1	MP 7
4.NBT.5 -2	A Multiply two two-digit numbers, using strategies based on place value and the properties of operations	i) Tasks do not have a context. ii) For the illustrate/explain aspect of 4.NBT.5, see 4.C.1.1	MP 7
4.NBT.6 -1	Find whole-number quotients and remainders with three-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division .	. i) Tasks do not have a context. ii) Tasks may include remainders of 0 in no more than 20% of the tasks. iii) For the illustrate/explain aspect of 4.NBT.6, see 4.C.1-2 and 4.C.2	MP 7,8
4.NBT.6 -2	Find whole-number quotients and remainders with four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division	. i) Tasks do not have a context. ii) Tasks may include remainders of 0 in no more than 20% of the tasks. iii) For the illustrate/explain aspect of 4.NBT.6, see 4.C.1-2 and 4.C.2	MP.7,8
4.NBT. Int.1	Perform computations by applying conceptual understanding of place value, rather than by applying multi-digit algorithms.		MP 1,7
4.C.1-1	Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 4.NBT.5	• Students need not use technical terms such as commutative, associative, distributive, or property. • Tasks do not have a context. • Unneeded parentheses should not be used. For example, use 4 + 3 x 2 rather than 4 + (3 x 2).	MP 3,6,7
4.C.1-2	Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 4.NBT.6	• Students need not use technical terms such as commutative, associative, distributive, or property. • Tasks do not have a context. • Unneeded parentheses should not be used. For example, use 4 + 3 x 2 rather than 4 + (3 x 2).	MP 3,6,7,8
4.C.2	Base explanations/reasoning on the relationship between multiplication and division. Content Scope: Knowledge and skills articulated in 4.NBT.6	• Tasks do not have a context.	MP 3,6,7

Number Talks

What does Number Talks look like?

- Students are near each other so they can communicate with each other (central meeting place)
- Students are mentally solving problems
- Students are given thinking time
- Thumbs up show when they are ready
- Teacher is recording students' thinking

Communication

- Having to talk out loud about a problem helps students clarify their own thinking
- Allow students to listen to other's strategies and value other's thinking
- Gives the teacher the opportunity to hear student's thinking

Mental Math

- When you are solving a problem mentally you must rely on what you know and understand about the numbers instead of memorized procedures
- You must be efficient when computing mentally because you can hold a lot of quantities in your head

Thumbs Up

- This is just a signal to let you know that you have given your students enough time to think about the problem
- If will give you a picture of who is able to compute mentally and who is struggling
- It isn't as distracting as a waving hand

Teacher as Recorder

- Allows you to record students' thinking in the correct notation
- Provides a visual to look at and refer back to
- Allows you to keep a record of the problems posed and which students offered specific strategies

Purposeful Problems

- Start with small numbers so the students can learn to focus on the strategies instead of getting lost in the numbers
- Use a number string (a string of problems that are related to and scaffold each other)

Starting Number Talks in your Classroom

- Start with specific problems in mind
- Be prepared to offer a strategy from a previous student
- It is ok to put a student's strategy on the backburner
- Limit your number talks to about 15 minutes
- Ask a question, don't tell!

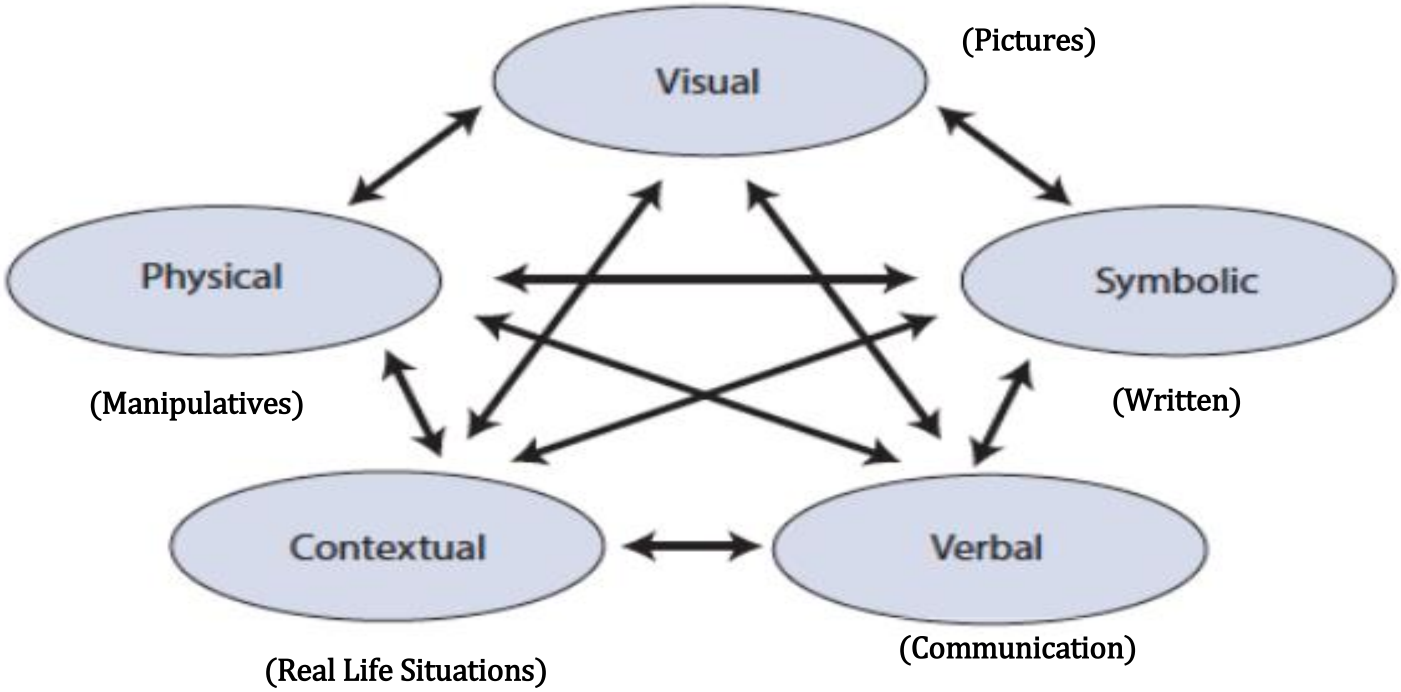
The teacher asks questions:

- Who would like to share their thinking?
- Who did it another way?
- How many people solved it the same way as Billy?
- Does anyone have any questions for Billy?
- Billy, can you tell us where you got that 5?
- How did you figure that out?

Student Name: ________________________________ Task: _______________________ School: ________ Teacher: ___________ Date: ________

"I CAN….."	STUDENT FRIENDLY RUBRIC				SCORE
	…a start 1	…getting there 2	…that's it 3	WOW! 4	
Understand	I need help.	I need some help.	I do not need help.	I can help a class-mate.	
Solve	I am unable to use a strategy.	I can start to use a strategy.	I can solve it more than one way.	I can use more than one strategy and talk about how they get to the same answer.	
Say or Write	I am unable to say or write.	I can write or say some of what I did.	I can write and talk about what I did. I can write or talk about why I did it.	I can write and say what I did and why I did it.	
Draw or Show	I am not able to draw or show my thinking.	I can draw, but not show my thinking; or I can show but not draw my thinking;	I can draw and show my thinking	I can draw, show and talk about my think-ing.	

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Teacher Questioning:

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26. **How did you begin** to think about this problem?
27. What is **another way** you could solve this problem?
28. How could you **prove** ______?
29. Can you **explain how your answer is different from or the same as** [student]'s answer?
30. Let's **break the problem into parts**. What would the parts be?
31. Can you **explain this part more specifically**?
32. Does that **always work**?
33. Can you think of a case where that **wouldn't work**?
34. How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35. What do you need to do **next**?
36. What have you **accomplished**?
37. What are your **strengths and weaknesses**?
38. Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39. What is this problem about? What can you **tell me about it**?
40. Do you need to **define or set limits** for the problem?
41. How would you **interpret** that?
42. Could you **reword that in simpler terms**?
43. Is there something that can be **eliminated** or that is **missing**?
44. Could you **explain** what the problem is asking?
45. What **assumptions** do you have to make?
46. What do you **know** about this part?
47. Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)
3.NBT.A.2: Add/subtract within 1000
4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm
5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

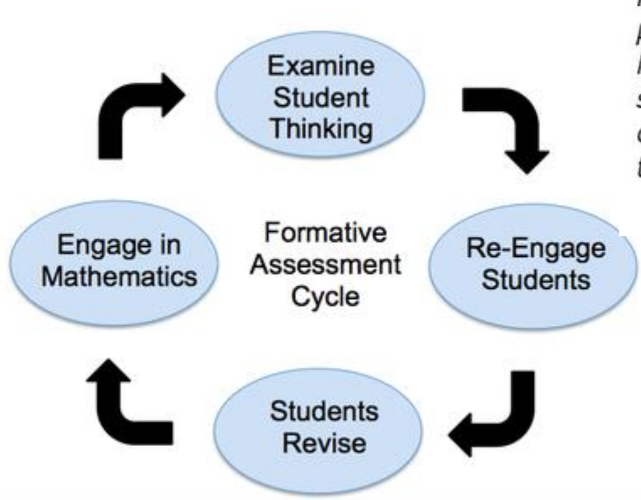
Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.



Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

The Standards for Mathematical Practice:

Describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1	**Make sense of problems and persevere in solving them** Mathematically proficient students in grade 4 know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Fourth graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try different approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively** Mathematically proficient fourth graders should recognize that a number represents a specific quantity. They connect the quantity to written symbols and create a logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities. They extend this understanding from whole numbers to their work with fractions and decimals. Students write simple expressions, record calculations with numbers, and represent or round numbers using place value concepts.
3	**Construct viable arguments and critique the reasoning of others** In fourth grade mathematically proficient students may construct arguments using concrete referents, such as objects, pictures, and drawings. They explain their thinking and make connections between models and equations. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics** Mathematically proficient fourth grade students experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fourth graders should evalu-

	ate their results in the context of the situation and reflect on whether the results make sense.
5	**Use appropriate tools strategically** Mathematically proficient fourth graders consider the available tools(including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use graph paper or a number line to represent and compare decimals and protractors to measure angles. They use other measurement tools to understand the relative size of units within a system and express measurements given in larger units in terms of smaller units.
6	**Attend to precision** As fourth graders develop their mathematical communication skills, they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, they use appropriate labels when creating a line plot.
7	**Look for and make use of structure** In fourth grade mathematically proficient students look closely to discover a pattern or structure. For instance, students use properties of operations to explain calculations (partial products model). They relate representations of counting problems such as tree diagrams and arrays to the multiplication principal of counting. They generate number or shape patterns that follow a given rule.
8	**Look for and express regularity in repeated reasoning** Students in fourth grade should notice repetitive actions in computation to make generalizations Students use models to explain calculations and understand how algorithms work. They also use models to examine patterns and generate their own algorithms. For example, students use visual fraction models to write equivalent fractions.

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

MATH CENTERS/ WORKSTATIONS

Math workstations allow students to engage in authentic and meaningful hands-on learning. They often last for several weeks, giving students time to reinforce or extend their prior instruction. Before students have an opportunity to use the materials in a station, introduce them to the whole class, several times. Once they have an understanding of the concept, the materials are then added to the work stations.

Station Organization and Management Sample

Teacher A has 12 containers labeled 1 to 12. The numbers correspond to the numbers on the rotation chart. She pairs students who can work well together, who have similar skills, and who need more practice on the same concepts or skills. Each day during math work stations, students use the center chart to see which box they will be using and who their partner will be. Everything they need for their station will be in their box. **Each station is differentiated**. If students need more practice and experience working on numbers 0 to 10, those will be the only numbers in their box. If they are ready to move on into the teens, then she will place higher number activities into the box for them to work with.

In the beginning there is a lot of prepping involved in gathering, creating, and organizing the work stations. However, once all of the initial work is complete, the stations are easy to manage. Many of her stations stay in rotation for three or four weeks to give students ample opportunity to master the skills and concepts.

Read *Math Work Stations* by Debbie Diller.
In her book, she leads you step-by-step through the process of implementing work stations.

MATH WORKSTATION INFORMATION CARD

Math Workstation: ______________________________________ Time: __________

NJSLS.:

____
______
____
______

Objective(s): By the end of this task, I will be able to:

- ____
- ____
- ____

Task(s):

- ____
- ____
- ____
- ____

Exit Ticket:

- ____
- ____
- ____

MATH WORKSTATION SCHEDULE

Week of: ________________

DAY	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal	Small Group Instruction
Mon.	Group ____	Group ____	Group ____	Group ____	BASED ON CURRENT OBSERVATIONAL DATA
Tues.	Group ____	Group ____	Group ____	Group ____	
Wed.	Group ____	Group ____	Group ____	Group ____	
Thurs.	Group ____	Group ____	Group ____	Group ____	
Fri.	Group ____	Group ____	Group ____	Group ____	

INSTRUCTIONAL GROUPING

	GROUP A		GROUP B
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
	GROUP C		GROUP D
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	

DATA DRIVEN INSTRUCTION

Formative assessments inform instructional decisions. Taking inventories and assessments, observing reading and writing behaviors, studying work samples and listening to student talk are essential components of gathering data. When we take notes, ask questions in a student conference, lean in while a student is working or utilize a more formal assessment we are gathering data. Learning how to take the data and record it in a meaningful way is the beginning of the cycle.

Analysis of the data is an important step in the process. What is this data telling us? We must look for patterns, as well as compare the notes we have taken with work samples and other assessments. We need to decide what are the strengths and needs of individuals, small groups of students and the entire class. Sometimes it helps to work with others at your grade level to analyze the data.

Once we have analyzed our data and created our findings, it is time to make informed instructional decisions. These decisions are guided by the following questions:

- What mathematical practice(s) and strategies will I utilize to teach to these needs?
- What sort of grouping will allow for the best opportunity for the students to learn what it is I see as a need?
- Will I teach these strategies to the whole class, in a small guided group or in an individual conference?
- Which method and grouping will be the most effective and efficient? What specific objective(s) will I be teaching?

Answering these questions will help inform instructional decisions and will influence lesson planning.

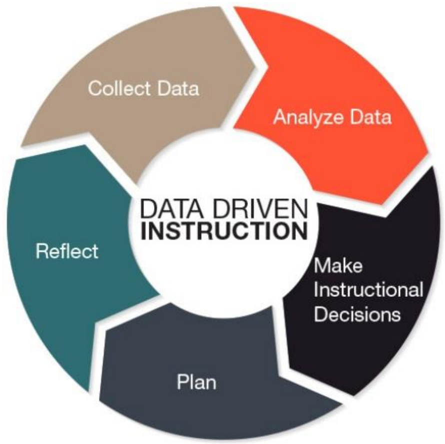
Then we create our instructional plan for the unit/month/week/day and specific lessons.

It's important now to reflect on what you have taught.

Did you observe evidence of student learning through your checks for understanding, and through direct application in student work?

What did you hear and see students doing in their reading and writing?

Now it is time to begin the analysis again.



Data Analysis Form School: ________________ Teacher: _______________________ Date: _____________
Assessment: _______________________________________ NJSLS: ____

GROUPS (STUDENT INITIALS)	SUPPORT PLAN	PROGRESS
MASTERED (86% - 100%) (PLD 4/5):		
DEVELOPING (67% - 85%) (PLD 3):		
INSECURE (51%-65%) (PLD 2):		
BEGINNING (0%-50%) (PLD 1):		

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MATH PORTFOLIO EXPECTATIONS

The Student Assessment Portfolios for Mathematics are used as a means of documenting and evaluating students' academic growth and development over time and in relation to the CCSS-M. The September task entry(-ies) should reflect the prior year content and *can serve* as an additional baseline measure.

All tasks contained within the **Student Assessment Portfolios** should be aligned to NJSLS and be "practice forward" (closely aligned to the Standards for Mathematical Practice).

Four (4) or more additional tasks will be included in the **Student Assessment Portfolios** for Student Reflection and will be labeled as such.

GENERAL PORTFOLIO EXPECTATIONS:

- Tasks contained within the Student Assessment Portfolios are "practice forward" and denoted as "Individual", "Partner/Group", and "Individual w/Opportunity for Student Interviews[1].
- Each Student Assessment Portfolio should contain a "Task Log" that documents all tasks, standards, and rubric scores aligned to the performance level descriptors (PLDs).
- Student work should be attached to a completed rubric; with appropriate teacher feedback on student work.
- Students will have multiple opportunities to revisit certain standards. Teachers will capture each additional opportunity "as a new and separate score" in the task log.
- A 2-pocket folder for each Student Assessment Portfolio is *recommended.*
- All Student Assessment Portfolio entries should be scored and recorded as an Authentic Assessment grade (25%)[2].
- All Student Assessment Portfolios must be clearly labeled, maintained for all students, inclusive of constructive teacher and student feedback and accessible for review.

4TH Grade Authentic Performance Task #1: Minutes and Days

Name:______________________________________

Jillian says

I know that 20 times 7 is 140 and if I take away 2 sevens that leaves 126. So 126 ÷ 7 = 18.

- ***a.*** Is Jillian's calculation correct? Explain.
- b. Draw a picture showing Jillian's reasoning.
- c. Use Jillian's method to find 222 ÷ 6.

Minutes and Days

NJSLS.MATH.CONTENT 4.NBT.6

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Mathematical Practices: 3, 4, and 6

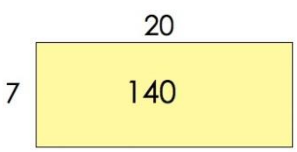
SOLUTION:

a. Jillian's reasoning is correct. She has found 20×7=140 and2×7=14. This means that

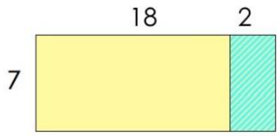
$$18\times7=(20-2)\times7=(20\times7)-(2\times7)=140-14=126.$$

The second equality uses the distributive property. These equations tell us that 126 ÷ 7 = 18.

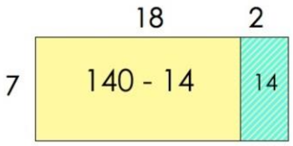
b. Jillian's initial idea of dividing 140 by 7 is represented here:



From there, Jillian decomposes the 20 sevens into 18 sevens and 2 sevens:



Lastly, Jillian recognizes that if the area of both rectangles combined would be 140, then she must subtract off the 2 extra sevens she used to get 140:



c.We have 40×6=240 and 3×6=18. So

$$37\times6=(40-3)\times6=(40\times6)-(3\times6)=240-18=222.$$

The second line uses the distributive property of multiplication.

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command

Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using : • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes an **efficient** and logical progression of steps.	Student correctly answers, clearly constructs, and communicates a complete response containing one minor calculation error based on explanations/reasoning using: • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes a **logical** progression of steps	Student correctly answers only two parts, clearly constructs, and communicates a response containing calculation and/or conceptual errors based on explanations/reasoning using: • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Student correctly answers only one part, clearly constructs, and communicates a response containing major calculation and/or conceptual errors based on explanations/reasoning using: • Strategies based on place value, the properties of operations, and/or the relationship between multiplication and division Response includes an **incomplete or Illogical** progression of steps.	The student shows no work or justification.

4^{TH} Grade Authentic Performance Task #1: Thousand and Millions of 4^{th} Graders

Name:______________________________________

There are almost 40 thousand fourth graders in Mississippi and almost 400 thousand fourth graders in Texas. There are almost 4 million fourth graders in the United States.

We write 4 million as 4,000,000. How many times more fourth graders are there in Texas than in Mississippi? How many times more fourth graders are there in the United States than in Texas? Use the approximate populations listed above to solve.

There are about 4 thousand fourth graders in Washington, D.C. How many times more fourth graders are there in the United States than in Washington, D.C.?

Performance Task Scoring Rubric:
Thousands and Millions of Fourth Graders

NJSLS.MATH.CONTENT.4.NBT.1

Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700÷70=10 by applying concepts of place value and division.

NJSLS.MATH.CONTENT.4.OA.1

Interpret a multiplication equation as a comparison, e.g., interpret 35=5×7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

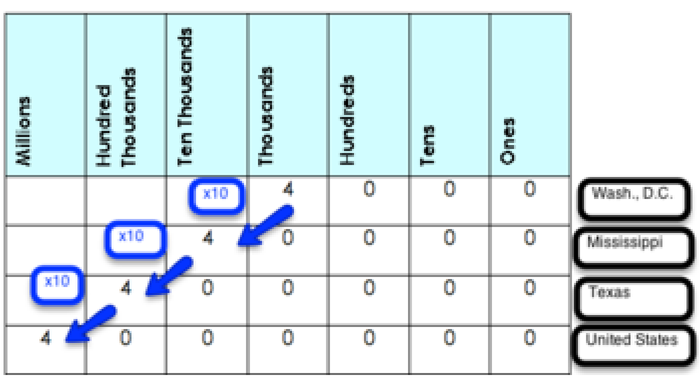
NJSLS.MATH.CONTENT.4.NBT.5

Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Mathematical Practices: 2, and 6

SOLUTION:

We write 4 thousand as 4,000
We write 40 thousand as 40,000
We write 400 thousand as 400,000
The value of each place is ten times the value of the place immediately to the right.



So:
40,000 is 10 times 4,000
400,000 is 10 times 40,000.
4,000,000 is 10 times 400,000.

Thus, 400,000 = 10 × 40,000, and there are about 10 times as many fourth graders in Texas as there are in Mississippi.
Also, 4,000,000 = 10 × 400,000, and there are about 10 times as many fourth graders in the US as there are in Texas.

Finally, to go from 4,000 to 4,000,000, we have to multiply by 10 three times. We see that

$$10\times10\times10=10\times100=1000$$

So there are about 1,000 times as many fourth graders in the US as there are in Washington DC.

Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using : • Concepts of place value and division • Strategies based on place value and the properties of operations Response includes an **efficient** and logical progression of steps.	Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using: • Concepts of place value and division • Strategies based on place value and the properties of operations. Response includes a **logical** progression of steps	Student answers, clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using: • Concepts of place value and division • Strategies based on place value and the properties of operations Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	Student answers, clearly constructs, and communicates a complete response with major calculation errors and/or conceptual errors based on explanations/reasoning using: • Concepts of place value and division • Strategies based on place value and the properties of operations Response includes an **incomplete or Illogical** progression of steps.	The student shows no work or justification.

Resources

Great Minds
https://greatminds.org/

Embarc
https://embarc.online/

Engage NY
http://www.engageny.org/video-library?f[0]=im_field_subject%3A19

Common Core Tools
http://commoncoretools.me/
http://www.ccsstoolbox.com/
http://www.achievethecore.org/steal-these-tools

Achieve the Core
http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

Manipulatives
http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000

http://www.thinkingblocks.com/

Illustrative Math Project :http://illustrativemathematics.org/standards/k8

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

Sample Balance Math Tasks: http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/

Georgia Department of Education:https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx

Gates Foundations Tasks:http://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf

Minnesota STEM Teachers' Center:
http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships

Singapore Math Tests K-12: http://www.misskoh.com

Mobymax.com: http://www.mobymax.com